MEASUREMENT OF THE COST-OF-LIVING INDEX IN THE EASI MODEL: EVIDENCE FROM THE JAPANESE EXPENDITURE DATA

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ABSTRACT

In this study, I measure the change in the cost-of-living index in Japan over approximately 20 years using newly developed demand system model. I find that the cost-of-living index, itself, was shifted upward by price changes, despite the existence of a protracted deflation in Japan. This implies an increase in consumer surplus along with changes in consumer expenditures, prices, and demographics. Further, I find that the substitution effect in the cost-of-living index is so large that the increase in consumer surplus depends on it. That is, consumers substitute greatly for cross-price goods in the face of price changes.

Contribution/ Originality: This study uses the new demand model by developed by Lewbel and Pendakur (2009) and measures changes in the cost-of-living index in Japan over approximately 20 years, from 1989 to 2011. I find that the cost-of-living index was shifted upward despite long-term deflation in Japan.

1. INTRODUCTION

In 2012, the Japanese Prime Minister Shinzo Abe published 'breakaway from deflation' as one of his policy objectives under 'Abenomics[1]', establishing the goal of a 2% per year price rise.[2] This policy objective will be assumed as one of the key objectives leading to the future recovery of the Japanese economy. For over 20 years, the Japanese economy suffered from the deflation, which

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[1] The Abenomics is a popular name given for a series of economic policies in Japan that politician, Shinzo Abe of the Liberal Democratic Party advocated in the second Abe Cabinet.

[2] Price denotes the consumer price index (CPI) from the Statistics Bureau.

ultimately led to a deflationary spiral. We presume that this negatively affected household budgets through price changes and the sluggish growth of household incomes. To capture the influence of these price changes, we measure the cost-of-living index according to price changes and price elasticities, and their fluctuations, using time-series data from over 20 years from 1989-2011. In our analysis, we use semi-macro panel data from 47 prefectural capitals. The panel data includes both a time-series dimension and a cross-sectional dimension and can measure the cost-of-living index considering the different budgets of each prefectural capital.

Estimating the cost-of-living index could reveal the practical impact of these price changes on Japanese households. Essentially, the argument for a cost-of-living index is that it implement for the efficacy of policy as the inflation target, based on demand analysis. The cost-of-living index, which is defined as the ratio of the minimum cost to achieve a base period level of utility at current period prices to the base period level of expenditure, incorporates substitution effects, expenditure levels, and household characteristics, and provides information about particular household circumstances, such as the impact of inflation. Measuring changes in the cost-of-living index poses many challenges because its construction for a group or society involves several issues of aggregation, substitution, equivalence scales, and welfare comparisons. However, there is much help available in the literatures for resolving these issues, such as that provided by Deaton and Muellbauer (1980); Fry and Parshardes (1989); Lewbel (1989) and Pollak (1989). For example, Fry and Parshardes (1989) constructed a true cost-of-living index of the Price Independence Generalized LOGarithmic (PIGLOG) model by modelling substitution as an aggregation shift parameter and exploiting the Tornqvist index. This index is expressed in terms of substitution effects and expenditure levels and includes demographic information and household characteristics.

The constitution of this paper is organized as follows. In Section 2, we introduce the Exact Affine Stone Index (EASI) demand model developed by Lewbel and Pendakur (2009) and the price effects derived from this model. In Section 3, we explain the data sources used in this study, and in Section 4, we report the empirical estimation results such as the compensated semi-elasticity calculation. In Section 5, we calculate the cost-of-living index in Japan for price changes during 1989 to 2011.

2. THE MODEL

2.1. The Exact Affine Stone Index Demand Model

In this analysis, we use the Exact Affine Stone Index (EASI) demand model developed by Lewbel and Pendakur (2009). The use of this model has valuable advantages over other demand systems in our analysis. First, since the EASI demand functions are derived from a specified cost function, given estimated parameters, it is possible to provide an expression for consumer surplus, such as a cost-of-living index for price changes. Second, the EASI model can be polynomials of any order in log real expenditure y and can have any rank up to $N - 1$, where N is the number of goods, although we have always believed that the limit of rank is three even with polynomial

Engel curves by Gorman (1981).[3] In Section 4, we will show that the Engel curve in Japanese households does not necessarily hold linear relationships between budget shares w and log expenditure x, and further, we observe high-dimensional relationships between them in the specific goods. Therefore, using the EASI model leads us to perform a further fitted model in the estimation.

As the beginning derivation of this empirical model, Lewbel and Pendakur (2009) define the specified cost function that accommodates high-rank Engel curves as:

$$C(\boldsymbol{p}, u, \boldsymbol{z}, \boldsymbol{\varepsilon}) = u + \sum_{i=1}^{N} \ln p_i \left[\sum_{r=0}^{N-1} b_{rj} u^r + \sum_{l=1}^{L} (C_{lj} z_l + D_{lj} z_l u) \right]$$

$$+ \frac{1}{2} \sum_{l=0}^{L} \sum_{i=1}^{N} A_{lij} z_l \ln p_i^2 + \frac{1}{2} \sum_{i=1}^{N} B_{ij} \ln p_i^2 u + \sum_{i=1}^{N} \ln p_i \varepsilon_i .$$

$$i, j = 1, \dots, N, \qquad l = 1, \dots, L, \qquad r = 1, \dots, N-1. \qquad (1)$$

By Shephard's lemma, we obtain Hicksian (compensated) budget share functions as $w = \omega(\boldsymbol{p}, u, \boldsymbol{z}, \boldsymbol{\varepsilon})$. The jth budget share is given by

$$w_j = \sum_{r=0}^{N-1} b_{rj} u^r + \sum_{l=1}^{L} (C_{lj} z_l + D_{lj} z_l u) + \sum_{l=0}^{L} \sum_{i=1}^{N} A_{lij} z_l \ln p_i + \sum_{i=1}^{N} B_{ij} \ln p_i u + \varepsilon_j. \qquad (2)$$

The expression for u can be yielded by an implicit utility y:[4]

$$y = \frac{x - \sum_{j=1}^{N} \ln p_j w_j + \sum_{l=0}^{L} \sum_{i=1}^{N} A_{lij} z_l \ln p_k^2 / 2}{1 - \sum_{i=1}^{N} B_{ij} \ln p_i^2 / 2}. \qquad (3)$$

The implicit utility y of (3) can represent as the log real expenditure.[5]

By substituting implicit utility y into the Hicksian budget shares, we obtain the jth implicit Marshallian budget share w_{jht} for goods j by household h in period t as

$$w_{jht} = \sum_{r=0}^{N-1} b_{rj} y_{ht}^r + \sum_{l=1}^{L} (C_{lj} z_{lht} + D_{lj} z_{lht} y_{ht}) + \sum_{l=0}^{L} \sum_{i=1}^{N} A_{lij} z_{lht} \ln p_{iht} + \sum_{i=1}^{N} B_{ij} \ln p_{iht} y_{ht}$$

$$+ \varepsilon_{jht},$$
$$h = 1, \dots, H, \qquad t = 1, \dots, T, \qquad (4)$$

Where N is the number of goods in this system, w_j is the jth budget share in total expenditure, and $\ln p_i$ is the log price of goods i. C_{lj} and D_{lj} are the parameters for the observable demographics $\boldsymbol{z}$ and the interactions with the observable demographics $\boldsymbol{z}$ and with log real

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[3] For example, the Almost Ideal Demand System (AIDS) by Deaton and Muellbauer (1980). is a popular model with rank two and the Quadratic Almost Ideal Demand System (QUAIDS) models by Banks, Blundell and Lewbel (1997). is rank three.

[4] Indirect utility V is the inverse of log cost function with respect to $u = V(\boldsymbol{p}, x, \boldsymbol{z}, \varepsilon)$. If an analytical V is unavailable, we can express utility as a function $u = g(\boldsymbol{\omega}(\boldsymbol{p}, u, \varepsilon), \boldsymbol{p}, x, \boldsymbol{z})$. By construction, $y = u$ and the implicit Marshallian demand system is given by $\boldsymbol{w} = \boldsymbol{\omega}(\boldsymbol{p}, y, \boldsymbol{z}, \varepsilon)$, which is the Hicksian demand system except u is replaced with y.

[5] (3) equals log expenditure x in the base period if $p_0 = 1$. However in the Japanese demand system, we often use as $p_0 = 100$, so it does not equal to log expenditure.

expenditure y. A and B are coefficients for the interactions with the observable demographics $\boldsymbol{z}$ and the compensated price effects $\boldsymbol{p}$ and the interactions with real expenditure and with price effects. Unlike the AIDS and other models, the error terms $\boldsymbol{\varepsilon}$ of EASI budget shares equal unobserved preference heterogeneity or random utility parameters. That is, it is possible that the error terms have an economic interpretation. We call (4) as an implicit Marshallian EASI model. As described above, implicit Marshallian EASI demands have no rank restriction, as noted by Gorman (1981). In order to estimate parameters simultaneously with (3), we use three-stage least squares (3SLS) or generalized method of moments (GMM) estimation with instruments to account for endogeneity due to the appearance of the right-hand side of $\boldsymbol{w}$.[6]

Letting the deflator of nominal expenditure as a Stone (1954) price index yields the approximate EASI model. That is, if we replace real expenditure y of (3) with

$$y_{ht}^{*} = x - \sum_{j=1}^{N} \ln p_{jht} w_{jht}, \qquad (5)$$

We can estimate the approximate EASI model by ordinary least squares (OLS).[7] Lewbel and Pendakur (2009) showed there was little empirical difference between the exact nonlinear and approximate linear EASI model. In the AIDS model, we have already found that the approximate model has an empirical difference in estimates when compared with the exact model (Pashardes, 1993; Buse, 1994). Unlike the AIDS model, the use of a Stone index for nominal expenditure x in the EASI model is exact, and it is the correct deflator for obtaining real expenditure. However, we find that this use also yields endogeneity as the exact model. The EASI cost function (1) satisfies the general regularity of cost function. That is, the adding-up constraint is given by

$$\sum_{j=1}^{N} b_{0j} = 1, \sum_{j=1}^{N} b_{rj} = 0, \sum_{j=1}^{N} A_{lij} = \sum_{j=1}^{N} B_{ij} = 0, \sum_{j=1}^{N} C_{lj} = \sum_{j=1}^{N} D_{lj} = 0, \text{and} \sum_{j=1}^{N} \varepsilon_{jht} = 0,$$

$$\text{for } r = 1, \dots, N-1, \qquad j = 1, \dots, N, \qquad l = 1, \dots, L. \qquad (6)$$

Since the adding-up condition is automatically satisfied, we use the $N-1$ equations in the estimation. The homogeneity conditions are given by

$$\sum_{i=1}^{N} A_{lij} = \sum_{i=1}^{N} B_{ij} = 0. \qquad (7)$$

The symmetry of the $\boldsymbol{A}$ and $\boldsymbol{B}$ coefficients also ensures Slutsky symmetry as

$$A_{ij} = A_{ji} \text{ and } B_{ij} = B_{ji}. \qquad (8)$$

In addition, the EASI cost function also requires the strict monotonicity $\partial C(\boldsymbol{p}, u, \boldsymbol{z}, \boldsymbol{\varepsilon})/\partial u > 0$ and the concavity of $\exp[C(\boldsymbol{p}, u, \boldsymbol{z}, \boldsymbol{\varepsilon})]$. A sufficient condition for concavity is $\partial^2 C(\boldsymbol{p}, u, \boldsymbol{z}, \boldsymbol{\varepsilon})/\partial p_j^2 < 0$, as negative semi-definite.

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[6] However, Lewbel and Pendakur (2009). showed that the bias from ignoring endogeneity could be small in empirical analysis.

[7] Approximate EASI model nests the approximate AIDS model if $y = x - \sum_{j=1}^{N} p_j \bar{w}_j$, where $\bar{w}_j$ is the average budget share. And it also nests the QUAIDS model by Banks, Blundell and Lewbel (1997).

2.2. Semi-Elasticities and the Cost-Of-Living Index

We first consider semi-elasticities to be derivatives of budget share with respect to log real expenditure y and log prices $\boldsymbol{p}$, and expenditure elasticities with respect to log prices $\boldsymbol{p}$. The semi-elasticities with respect to log real expenditure y are given by

$$\theta_j = \frac{\partial \boldsymbol{w_j}}{\partial y} = \sum_{r=1}^{N-1} b_r r y^{r-1} + \sum_{l=1}^{L} D_{lj} z_l + \sum_{i=1}^{N} B_{ij} \ln p_i. \qquad (9)$$

This is possible to interpret as real expenditure elasticities. Compensated price semi-elasticities are given by

$$\mu_{ij} = \frac{\partial \boldsymbol{w_j}}{\partial \boldsymbol{p}} = \sum_{l=0}^{L} \sum_{i=1}^{N} A_{lij} z_l + \sum_{i=1}^{N} B_{ij} y. \qquad (10)$$

If we express (10) as matrix $\boldsymbol{\Gamma} \equiv \sum_l \boldsymbol{A_l} z_l + \boldsymbol{B} y$, compensated expenditure elasticities with respect to log prices $\boldsymbol{p}$ are given by

$$\mathrm{W}^{-1}(\Gamma + ww'), \qquad \text{where } \mathrm{W} = \mathrm{diag}(w). \qquad (11)$$

We next consider the evaluation of cost to a price change. We measure a consumer surplus for a price change from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$ by the cost function of (1) as follows:

$$C(\boldsymbol{p}_1, u, \boldsymbol{z}, \boldsymbol{\varepsilon}) - C(\boldsymbol{p}_0, u, \boldsymbol{z}, \boldsymbol{\varepsilon})$$

$$= (\boldsymbol{p}_1 - \boldsymbol{p}_0)'\boldsymbol{w}_0 + \frac{1}{2}(\boldsymbol{p}_1 - \boldsymbol{p}_0)'(\sum_{l=0}^{L} \sum_{i=1}^{N} A_{lij} z_l + \sum_{i=1}^{N} B_{ij} y)(\boldsymbol{p}_1 - \boldsymbol{p}_0).$$

(12)

This is named as the cost-of-living index. The cost-of-living index is defined as the ratio of the minimum expenditure required to attain the base preference at prices $\boldsymbol{p}_0$ to that required at prices $\boldsymbol{p}_1$. The first term of (12) expresses the Stone index $(\boldsymbol{p}_1 - \boldsymbol{p}_0)'\boldsymbol{w}_0$ for a price change from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$. That is, the effect in the first term is derived from the budget shares of the base preference $\boldsymbol{w}_0$ and unobserved heterogeneity $\boldsymbol{\varepsilon}$. The second term allows us to model substitution effects explicitly. The effect in the second term is derived from the quadratic of a price change from $\boldsymbol{p}_0$ to $\boldsymbol{p}_1$, observable demographics $\boldsymbol{z}$, and log real expenditure y.

The unobserved heterogeneity enters only through the first term $\boldsymbol{w}_0$. We can also think of the cost-of-living index as incorporating these two effects. The traditional nonparametric approaches to the cost-of-living index use only the first-order term, which accommodates unobserved heterogeneity, but does not incorporate the second-order term, which captures substitution effects. Our approach has the advantage of measuring these two effects in the cost-of-living index.

3. DATA AND SOURCES

The Household Survey data used in our analysis are the panel data for two groups – workers' households and other households – in 47 prefectural capitals. The source of data is the *Family Income and Expenditure Survey* ('Kakei Chosa' in Japanese) by the Japanese Statistics Bureau, from

1989 to 2011.[8] We classified the data into 10 goods: food, housing, fuel, furniture, clothing, medicine, transportation, education, recreation, and miscellaneous. The household survey also provides demographic information and household characteristics, such as age of the head of household, numbers of workers in a household and household size. Price series data are obtained from the Consumer Price Index (CPI) and are calculated by using 2005 as the base year.

Japanese household data shows the different characteristics among 47 prefectural capitals. In budget shares, for example, the Kanto metropolitan areas such as Tokyo, Chiba, and Kawasaki Cities have high budget shares for clothing and recreation, but hold low budget shares for miscellaneous. Alternatively, local cities such as Toyama and Fukushima have high budget shares for miscellaneous and low budget shares for clothing and recreation.

This difference between the cities in Japan would occur by the difference of traffic conditions and commercial facilities in a location. In addition, particularly big cities by the population, including the Kanto metropolitan areas and Osaka City, hold high budget shares for housing in comparison with the small-scale cities located in the district. Based on these features, we expect the presence of individual fixed effects, capturing the different characteristics among the 47 prefectural capitals.

On the other hand, our data series must include the time-oriented change of characteristics over approximately 20 years. Figures 1 and 2 show the mean budget shares and log prices over the period 1989-2011. In Figure 1, budget shares for housing, clothing and miscellaneous display a downward trend for approximately 20 years, whereas those for fuel, medicine, transport, and education, display an upward trend with price changes. In particular, the growth rate for transport is the largest among these commodities, capturing the rapid increase of communication expenses for Internet and mobile phone usage included under the transport category in recent years. We also expect the presence of fixed time effects, in keeping with price changes in Japanese consumer demand.

In addition, in Figure 2, log prices for housing and recreation display a downward trend. In particular, among the 10 commodities, the decrease in housing is the most remarkable, and this change would influence the decline of own-budget shares. By contrast, log prices for fuel, medicine, education, and miscellaneous display an upward trend. In particular, the increase in education is noteworthy, and this could be related to improvements in the level of education in Japan over the past 20 years. Later, we calculate the changes in the cost-of-living index over approximately 20 years, including these fixed effects.

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[8] We use unbalanced panel data containing partially missing values in other households from 1989 to 1999.
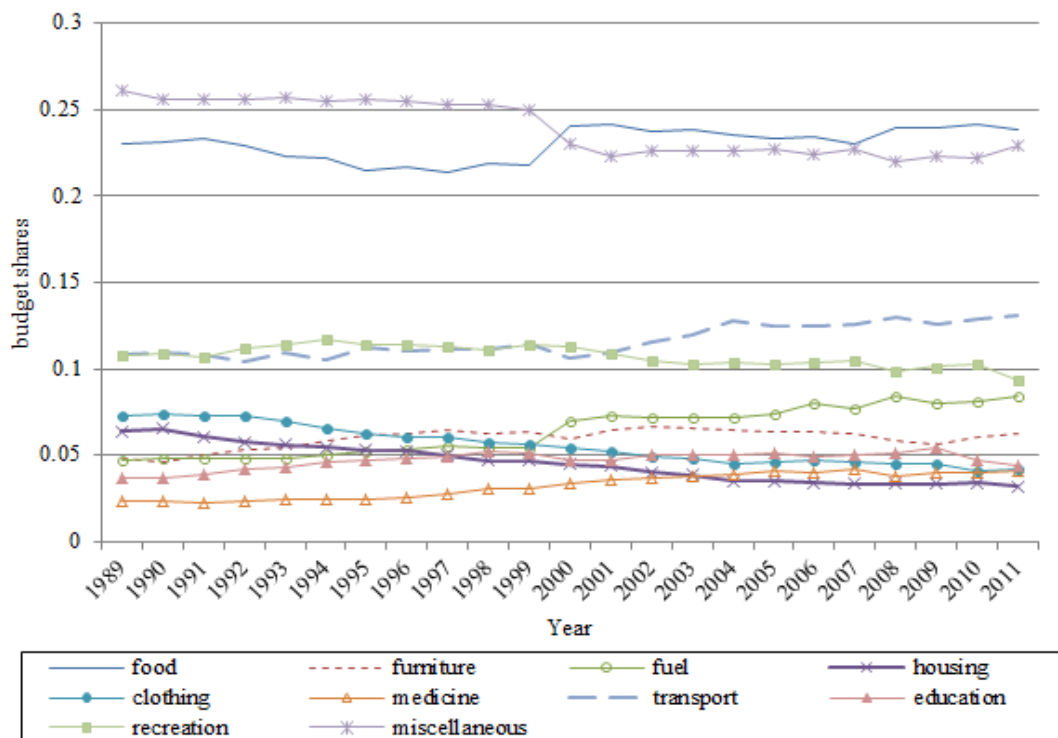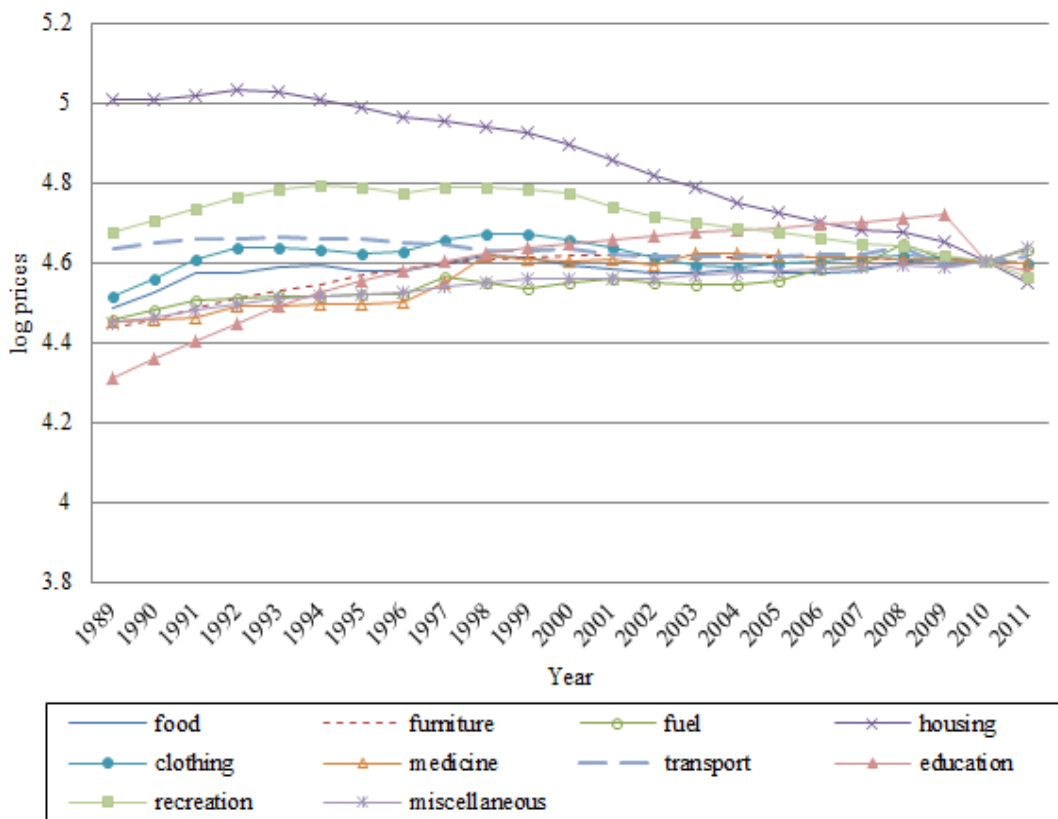
Figure-1. Budget shares from 1989-2011

Figure-2. Log prices from 1989-2011

Table 1. Model specification tests

Specifications	Null hypothesis	df	Test statistics	P-value
In order of log real expenditure y:				
y	$b_{1j} = 0$ for all j	9	30.902	0.0003
y^2	$b_{2j} = 0$ for all j	9	31.015	0.0003
y^3	$b_{3j} = 0$ for all j	9	31.119	0.0003
y, y^2, y^3	$b_{1j} = b_{2j} = b_{3j} = 0$ for all j	27	110.552	0.0000
In symmetry-restricted model with instruments in order of y^3:				
Specific effects				
(a) individual effect	$\alpha_{jh} = 0$ for all h	432	123.06	0.000
(b) time effect	$c_{jt} = 0$ for all t	162	328.429	0.000
(c) two-way fixed effect	$\alpha_{jh} = c_{jt} = 0$ for all h and t	594	4034.50	0.000
Price effect	$B = 0$	81	56.742	0.113
Price effect	$A_l = 0$ for all l	243	208.768	0.000
Price effect	$B = 0, A_l = 0$ for all l	324	516.145	0.000
Demographic effect	$D = 0$	27	122.005	0.000
Demographic effect	$C = 0, D = 0$	54	166.486	0.000
J test	over-identification for system	180	48.621	1.000
Sargan test	over-identification for food	54	52.434	0.535
	over-identification for housing	54	37.083	0.962
	over-identification for fuel	54	81.847	0.008
	over-identification for furniture	54	40.277	0.917
	over-identification for clothing	54	47.764	0.712
	over-identification for medicine	54	39.403	0.932
	over-identification for transport	54	16.964	0.999
	over-identification for education	54	37.339	0.959
	over-identification for recreation	54	31.706	0.993
Symmetry test	$A_l = A_l', B = B'$ for all l	216	173.196	0.628

4. EMPIRICAL ESTIMATION

4.1. Estimation of the EASI Model

Since the EASI cost function (1) satisfies the adding-up constraints, we estimate the system of $N - 1$ equations dropping the Nth last equation. The homogeneity constraint on parameters imposes the linear restrictions as in (7). The Slutsky symmetry of (8) is satisfied if and only if the **A** and **B** matrices are symmetric. Imposing symmetry restrictions requires the linear cross-equation equality restrictions on parameters and reduces the number of parameters. In line with Lewbel and Pendakur (2009) we use the parameters of the approximate EASI model with y_{ht}^* of (5) as the starting values for the exact model with y of (3) estimation. As described in Section 2.1, the complication of estimation for (4) is that budget shares **w** also appear on the right-hand side of the equation and this raises the endogeneity issue. In an econometric problem, it is obvious that ignoring this endogeneity results in estimation bias. In addition, we expect the possibility of heteroskedasticity for the error terms $\boldsymbol{\varepsilon}$.

To account for these problems in estimation, we use the instrumental variables estimators. We select lagged levels of the endogenous variables as instruments **q**, which is uncorrelated with unobserved heterogeneity $\boldsymbol{\varepsilon}$. Parameters can be estimated by applying, Hansen (1982) GMM to

this set of moments $E[\boldsymbol{\varepsilon}'\boldsymbol{q}] = 0$. If the Slutsky symmetry is not imposed on $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices, the model is exactly identified given instruments $\boldsymbol{q}$. In contrast, if the Slutsky symmetry is imposed on $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices, which have $(L + 2)(N - 1)(N - 2)/2$ restrictions, the model is over-identified given the instruments $\boldsymbol{q}$.

Furthermore, as described in Section 3, if there is unobserved heterogeneity, the error terms $\boldsymbol{\varepsilon}$ of (4) can be written as

$$\varepsilon_{jht} = \alpha_{jh} + c_{jt} + e_{jht}, \tag{13}$$
$$j = 1, \dots, N, \qquad h = 1, \dots, H, \qquad t = 1, \dots, T,$$

Where α_{jh} denotes an individual fixed effect and c_{jt} denotes a time fixed effect. And e_{jht} is assumed as strong exogeneity and $E(\boldsymbol{e}|\boldsymbol{\alpha}, \boldsymbol{c}, y^r, \boldsymbol{z}, \boldsymbol{zy}, \boldsymbol{p}, \boldsymbol{zp}, \boldsymbol{py}) = 0$. This formulation expresses persistent heterogeneity in the intercept of each equation. By (13), we assume the possible presence of unobserved heterogeneity in the estimation. When there is unobserved heterogeneity, we can identify the parameters of Equation (4) by further assumption with instruments. In our estimation, we assume the endogenous variables have a correlation with specific fixed effects. This assumption allows us to identify our model. In addition, our estimates rely on the following identifying assumptions: $E(e_{jht}y_{ht-s}^r) = 0$, $E(e_{jht}z_{jht}y_{ht-s}) = 0$, and $E(e_{jht}p_{jht}y_{ht-s}) = 0$ $(s = 1, \dots, S)$. Then, we are required to carry out the Sargan test of the over-identifying restrictions estimation as a test of whether the assumption of correlation between the endogenous variables and individual fixed effects is satisfied.

Table 1 shows Wald test results for the various hypotheses, so as to specify the model in this analysis.[9] First, we check for adequacy of our order polynomial in y. This result is indicated in the upper of Table 1. Individual hypothesis of exclusion for first-, second-, or third-order terms in y; $H_0: b_{1j} = 0$, $H_0: b_{2j} = 0$, or $H_0: b_{3j} = 0$ $(j = 1, \dots, N - 1)$, are all rejected at the 5% level significantly. The combination hypothesis of exclusions for first- to, third-order terms in y; $H_0: b_{1j} = b_{2j} = b_{3j} = 0$, is also rejected. Thus, we statistically confirm the sufficiency of a third-order polynomial y^3 in our model.

Next, we test the significance of the individual fixed effects and time fixed effects included in the intercept of each equation. We test them based on the symmetry-restricted model. These results are indicated in the middle section of Table 1. Our results show the significance of each individual fixed effect, or time fixed effect, and both individual fixed effects and time fixed effects. That is, we confirm the sufficiency of two-way fixed effects in our model. As described in Section 3, this result indicates the presence of the different characteristics between the 47 prefectural capitals and the transition over approximately 20 years. Furthermore, we test the significance of the $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ matrices in the specified model. The exclusion tests for these matrices show the significances of the direct price effects $\boldsymbol{A_0}$ (the third terms in (4)), the interactions between prices and demographics in $\boldsymbol{A}$, the interactions of prices with log real expenditure in $\boldsymbol{B}$, the direct

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[9] The Breusch and Pagan test results for heteroskedasticity in regression disturbances indicates that variances are not constant across observations. Under the heteroskedasticity for disturbance terms, we use the White (1980). estimator with heteroskedasticity-consistent covariance matrix instead.

demographic effects in C, and the interactions between demographics and log real expenditure in D.

In addition, we conduct the J test or the Sargan test of over-identifying restrictions for instrument validity and show the significance of instruments. As described above, we use the following the lagged levels of the endogenous variables for the equation as instruments in the GMM estimation: y^r_{t-s}, zy_{t-s}, and py_{t-s} ($s = 1, ..., 4$). We test whether the instruments are valid in the sense that they are uncorrelated with error term ε. In the J test for over-identification, we need the 216 symmetry restrictions for the system to have more moments than parameters in order to pass the over-identification test of instrument validity. The J test result shows the acceptance of over-identification restrictions in the model. Then, we show the Sargan test results for over-identification in each equation.

Table-2. Estimated compensated price effects and expenditure effects

	Food	Housing	Fuel	Furniture	Clothing	Medicine	Transport	Education	Recreation	miscellaneous
Expenditure semi-elasticity:										
	-0.2395	0.3251	-0.2055	-0.1278	-0.0150	-0.0272	0.1720	-0.0951	0.6652	0.7621
	(0.011)	(0.008)	(0.005)	(0.003)	(0.003)	(0.005)	(0.007)	(0.005)	(0.005)	(0.003)
Budget share semi-elasticity:										
Food	-3.2197	-13.2495	-12.1889	-10.0573	-6.7884	-9.1106	-15.0201	-13.9094	-13.7275	15.3722
	(0.011)	(0.041)	(0.042)	(0.039)	(0.025)	(0.031)	(0.049)	(0.046)	(0.042)	(0.006)
Housing		-10.1468	-7.3691	-5.5266	-5.4668	-7.0126	-3.4205	-0.7647	-2.7142	3.0404
		(0.026)	(0.011)	(0.006)	(0.008)	(0.014)	(0.016)	(0.010)	(0.010)	(0.001)
Fuel			5.0592	4.3903	3.7205	1.9430	4.3217	4.1616	3.4136	-3.8219
			(0.008)	(0.007)	(0.006)	(0.005)	(0.003)	(0.003)	(0.001)	(0.002)
Furniture				3.1850	2.8826	3.0631	-3.6187	2.9128	2.4895	-2.7875
				(0.005)	(0.004)	(0.005)	(0.003)	(0.002)	(0.001)	(0.001)
Clothing					-1.7367	1.1258	1.2994	1.1654	1.2043	-1.3474
					(0.006)	(0.007)	(0.008)	(0.008)	(0.009)	(0.001)
Medicine						-4.7268	-5.6889	-5.2028	-4.8113	5.3861
						(0.022)	(0.026)	(0.023)	(0.023)	(0.002)
Transport							3.2812	3.0087	3.8336	-4.2867
							(0.017)	(0.014)	(0.016)	(0.001)
Education								1.3907	-1.2983	-1.4524
								(0.017)	(0.023)	(0.0004)
Recreation									-1.6205	1.8101
									(0.007)	(0.001)
miscellaneous										-11.9129
										(0.005)
Own-price expenditure elasticity:										
	1.0464	-0.7871	1.6299	2.0526	0.4506	-1.4321	1.0309	-1.4233	0.8761	0.9104
	(0.008)	(0.026)	(0.018)	(0.018)	(0.023)	(0.042)	(0.011)	(0.027)	(0.012)	(0.098)

Note: The values in parentheses are asymptotic standard errors computed via the delta method.

The Sargan test rejects over-identification restrictions for only fuel, but does not reject the set of instruments for most equations. This provides evidence for the assumption of the correlation between observed variables and fixed effects.

Finally, the lower section of Table 1 shows the Slutsky symmetry tests in the interactions between prices with demographics in the A matrices and the interactions of prices with log real expenditure in the B matrices. The combination test results for both the level of prices ($A = A'$) and of prices interacting with implicit utility y ($B = B'$) do not reject symmetry at the 5% level of significance. This result suggests that the Slutsky symmetry is not violated and that the third-order polynomial in y is sufficient under the symmetry-restricted model.

4.2. Estimated Elasticities in the EASI Model

Table 2 presents the summary of estimated expenditure and price effects of symmetry-restricted results from GMM estimates with instruments. First, we report compensated semi-elasticities with respect to log real expenditure y in the first part of Table 2, with asymptotic standard errors computed via the delta method. Semi-elasticities with respect to log real expenditure y can be easily estimated by (9), interpreted as expenditure elasticity. We find that almost all have statistically significant compensated semi-elasticities at the 5% level. Expenditure elasticities for food, fuel, furniture, clothing, medicine, and education are negative and belong to inferior goods. For example, the expenditure elasticity for food is -0.2395 and implies that a 1% increase of expenditure for food is associated with about a 0.24 % point decrease of budget share for food. On the other hand, the expenditure elasticities for housing, transport, recreation, and miscellaneous are positive and belong to normal goods. In particular, the semi-elasticity for miscellaneous is 0.7621 and implies that a 1% increase of expenditure for miscellaneous is associated with a 0.76% point increase of budget share for recreation.

Next, the second block of Table 2 reports compensated price semi-elasticities. Compensated budget-share semi-elasticities with respect to prices p are given by (10). The own-price effects are statistically significant and relatively large. For example, the compensated own-price semi-elasticity for fuel is 5.0592, and implies that a 1% increase of price for fuel is associated with a 5.06 % point increase of budget share for fuel when expenditure is raised to hold utility constant. In contrast, the compensated own-price semi-elasticity for medicine is -4.7268. If the price for medicine increases by 1 %, its budget-share decreases by 4.73 % points. Further, we find cross-price effects are also large and statistically significant. This will suggest the importance of substitute on effects. In particular, the cross-price semi-elasticity for food against transport price is -15.0201. This implies that the increase in transport price is associated with a decrease of 15.02% points in the budget share for food. The budget share for food largely decreases for an increase in the price of all cross-price goods, and the substitution effects are large. This tendency also accommodates for housing, medicine, and miscellaneous. In contrast, for fuel, furniture, and transport, budget shares increase for an increase in cross-price goods.

Finally, the third part of Table 2 shows the own-price expenditure elasticities, given by (11). In (11), we enter w with the unobserved heterogeneity terms ε. We find that almost all have statistically significant own-price expenditure elasticities. Some of the own-price expenditure elasticities are positive. In particular, the own-price expenditure elasticity for fuel is 1.6299 and the elasticity for furniture is 2.0526, and these are highly positively elastic. In contrast, compensated medicine expenditures are negatively elastic, with a significant own-price expenditure elasticity of -1.4321.

In Table 2, we find that some of the own-price elasticity and semi-elasticity are significantly positive. This does not imply the absence of concavity immediately, but it is difficult to satisfy with the negativity of own-price elasticity in our system. On the other hand, the significance of own-price elasticity and semi-elasticity is obvious and the various price effects are also observed to be statistically significant.
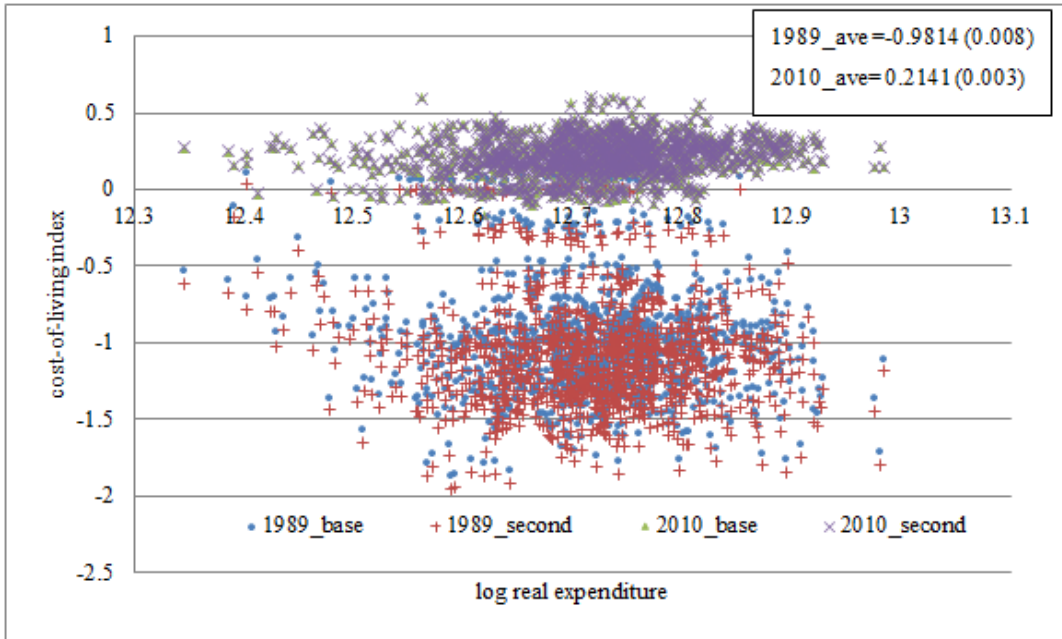
Figure-3. Cost-of-living index for equation systems in base preferences of 1989 and 2010
Note: In the upper box, the values in parentheses are standard errors.

5. MEASUREMENT OF THE COST-OF-LIVING INDEX IN JAPANESE EXPENDITURE DATA

5.1. Estimated Results by Equation Systems

In Figure 3, we measure the change in the cost-of-living index in Japan by comparing two indices estimated for two base preferences at prices p_0 in 1989 and 2010. The cost-of-living index includes the unobserved heterogeneity ε in w_0 and the substitution effects in (12). Figure 3 shows that the two cost-of-living indices are calculated with the base years of 1989 and 2010 and are divided into two layers. We find the indices moves up when the base year shifts from 1989 through 2010. The average cost-of-living index in the case of the base preference as 1989 is -0.9814 and as 2010 is 0.2141. That is, we can observe the increase of consumer surplus because of changing consumption expenditures, prices, and demographics. For the base year as 1989, the calculated index is negative and has a large bound deviation as compared with the case of 2010. In particular, the range from 12.6 to 12.8 in log real expenditure has a large scattered impact on the cost-of-living index. On the other hand, when the base year is 2010, the deviation of indices for every range in log real expenditure is small and it is frequently distributed around the average value. That is, the cost-of-living index holds a similar trend for every range in log real expenditure and does not incline to some range. Figure 1 also represents their substitution effects across the expenditure share equation captured by the second-order term of (12). We find that the second-order terms calculated in the two base years are large together and the cost-of-living index would depend on the trend of the second-order terms. That is, households substitute greatly for cross-price goods in the face of price changes. This effect is confirmed by the large substitution price effects as shown in Table 2.

5.2. Estimated Results by Goods

In Figures 4.1 to 4.10 display the cost-of-living index for 10 goods. Above all, we report the cost-of-living index for food, transport, and education, which show a characteristic change in comparison to the two cost-of-living indices. First, Figure 4.1 shows the change in the cost-of-living index for food. It shows a different trend than that between the two indices and shifts downwards when the base year changes from 1989 through 2010. The average cost-of-living index calculated with the base year as 1989 is 0.0059 and with the base year as 2010 is -0.0064. Unlike the increase in the overall cost for equation systems, as shown in Figure 3, the average cost for food decreases from 1989 through 2010. In the base year as 1989, the cost-of-living index is closely located around the average and seems to be decreasing as log real expenditure rises. However, with the base year as 2010, the cost-of-living index shows low and negative values and a large scattering particularly in the range from 12.6 to 12.8 in log real expenditure. By the change of base year from 1989 through 2010, the cost-of-living index, itself, observes a large bound deviation, but this is due to substitution effects rather than the change of budget shares of the base preference and unobserved heterogeneity. With the base year as 1989, the second-order terms were negatively small and not consistent with the change in the cost-of-living index. The cost-of-living index for food could be roughly expressed by the first-order term in the consumer surplus calculation, namely the Stone index for price change. On the other hand, in the base year as 2010, the second-order terms become large and consumers would substitute for cross-price goods in the face of price changes.

Second, Figure 4.7 shows the cost-of-living index for transport. Contrary to food, this index shifts upward when the base year changes from 1989 through 2010. In Japan, the recent rapid spread of consumer expenditure for mobile phones and the Internet has driven up the budget share for transport in 10 goods. Further, the cost of transport, itself, may affect the increase in costs in the overall system. The average cost-of-living index with the base year as 1989 is 0.0004 and with the base year as 2010 is 0.0046. We find that the cost-of-living index for transport shows a similar trend between the two indices. For example, in the range from 12.6 to 12.8 of log real expenditure, the cost-of-living index has a higher value and a large scattering. On the other hand, in other ranges of low and high, it is roughly distributed around the average value. That is, we find that the cost of transport is higher in the middle-range in log real expenditure. In transport, the second-order terms are large and consistent with the trend of their cost-of-living index in both cases of 1989 and 2010. That is, almost all can be expressed in terms of the substitution effects. We find that consumers substitute greatly for cross-price goods in the face of price changes.

Third, Figure 4.8 shows the cost-of-living index for education. As with food, the index shifts downward from 1989 through 2010. With the base year as 1989, the cost-of-living index is sparsely distributed in all ranges in log real expenditure and has a large scattering. With the base year as 2010, the index is approximately distributed around the average value and the cost of education falls in all ranges. In education, the substitution effects are large and almost all of the cost-of-living index is expressed in terms of these effects. The downward shifts of the cost-of-

living index are due to a decrease in the substitution effects rather than a change of budget shares for education in price change.

Finally, we obtain the following results for the remaining seven goods not yet reported in this sub-section. In fuel and furniture, we find that the cost-of-living index shifts downwards from 1989 through 2010. Above all, the cost-of-living index for fuel tends to be distributed around zero, and differences in all ranges of log real expenditure are small. On the other hand, housing, clothing, medicine, recreation, and miscellaneous shift upward in the cost-of-living index from 1989 through 2010. As a whole, the deviation of the cost-of-living index is small in 2010 as compared with 1989. This indicate that differences are small for every range of log real expenditure in the case of 2010, and the change for these goods does not depend on log real expenditure but price change.

We also estimated the cost-of-living indices with unobserved heterogeneity set to zero ($\varepsilon = 0$) although we do not report them in this paper. As shown in (12), the first term of the cost-of-living index includes the unobserved heterogeneity in $\boldsymbol{w}$. If we use $\hat{\boldsymbol{w}}$, which contains no error term, the unobserved heterogeneity component is not included in the cost-of-living index. We have already found that our estimates would remain almost unchanged if the errors were interpreted as ordinary error terms. In the cost-of-living index, we find that the impact of the treatment of ε as a measurement error is relatively small. That is, the treatment of ε would not greatly affect consumer surplus measures[10]. As described above, the effects in the second-order term are large and the cost-of-living index depends on these terms to a fair extent in our analysis.

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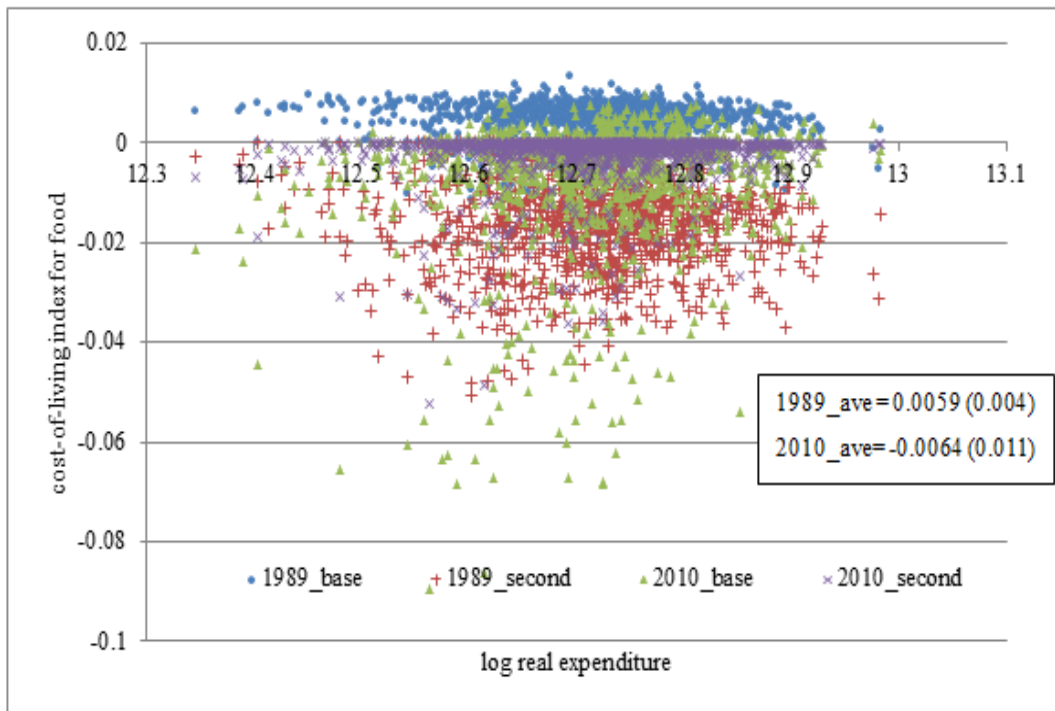
[10] However, the individual and time fixed effects are statistically significant as shown in Section 4.1 and our results should note that it is not an issue to deny the fixed effects included in (13).

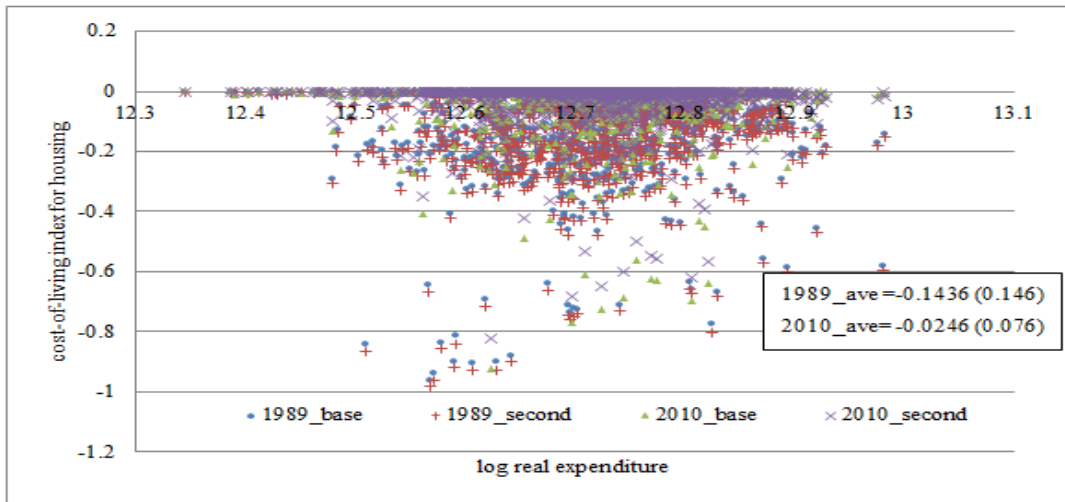**Figure-4.1.** Cost-of-living index for food in base preferences of 1989 and 2010

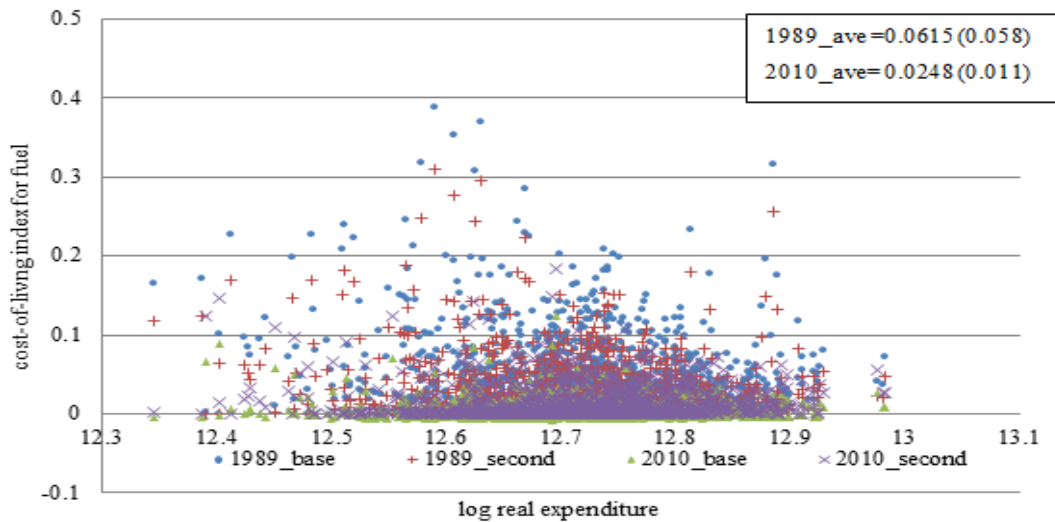**Figure-4.2**. Cost-of-living index for housing in base preferences of 1989 and 2010

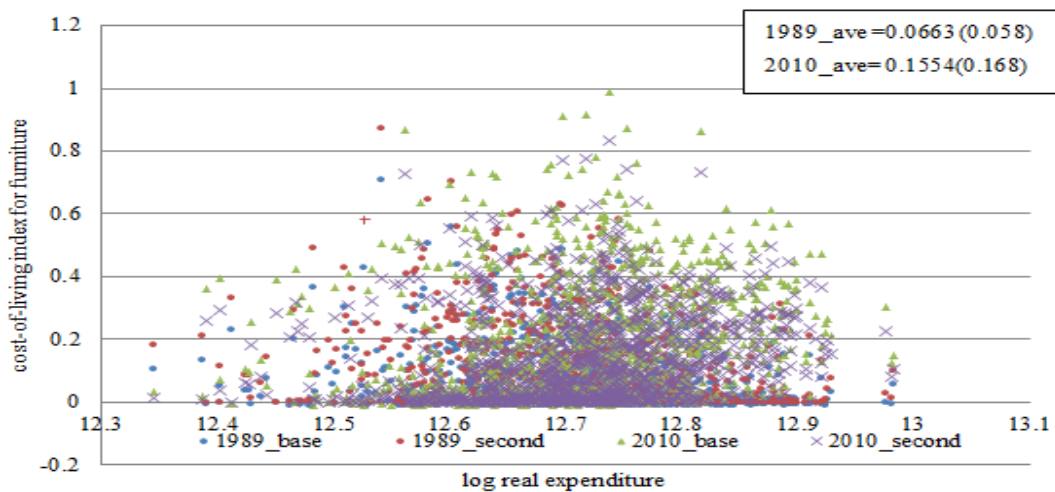**Figure-4.3.** Cost-of-living index for fuel in base preferences of 1989 and 2010

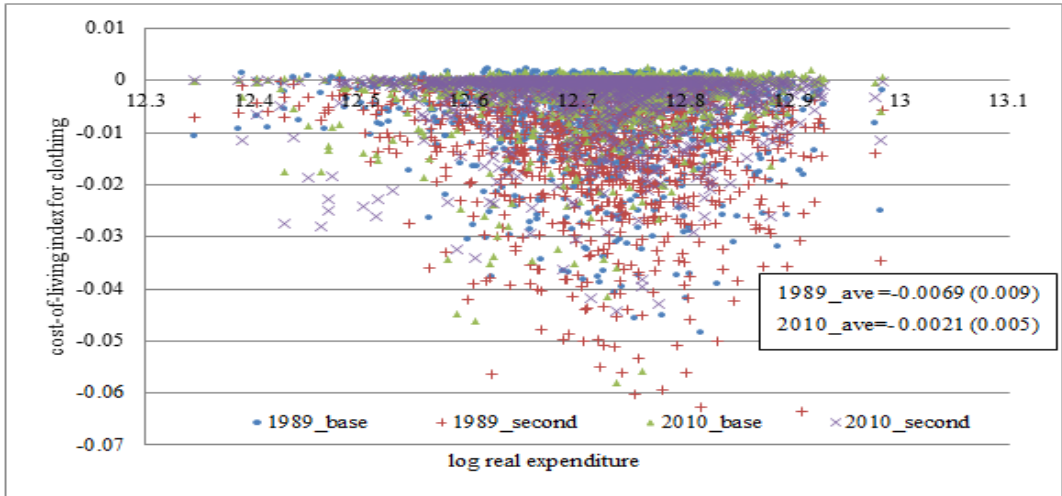**Figure-4.4.** Cost-of-living index for furniture in base preferences of 1989 and 2010

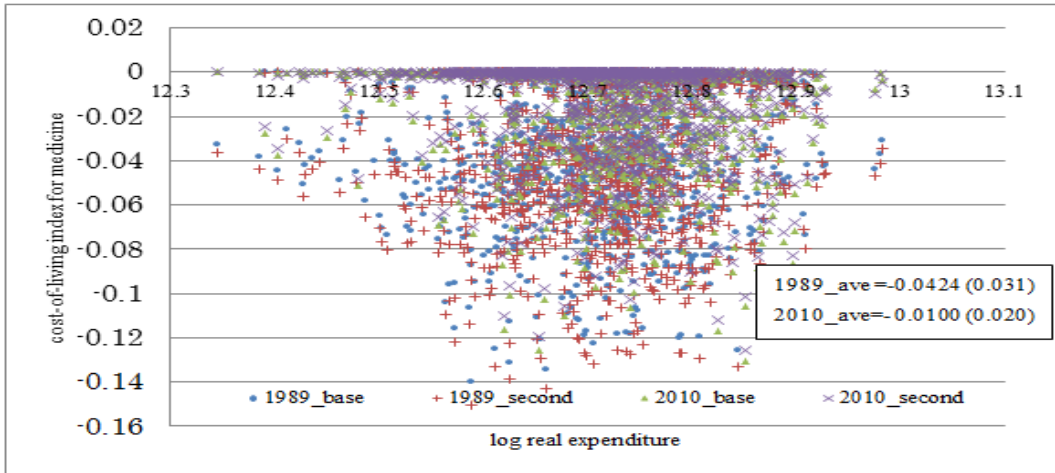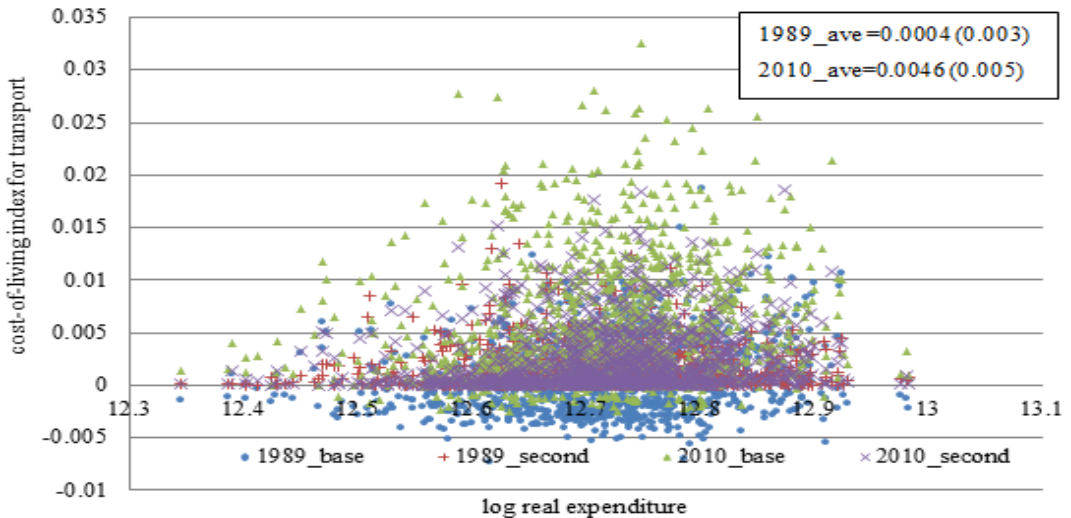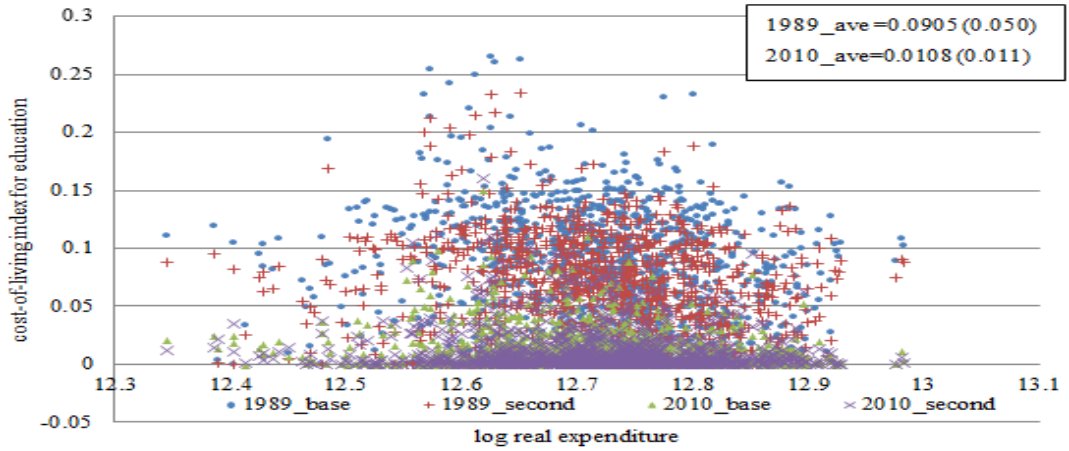**Figure-4.5.** Cost-of-living index for clothing in base preferences of 1989 and 2010

Figure-4.6. Cost-of-living index for medicine in base preferences of 1989 and 2010

Figure-4.7. Cost-of-living index for transport in base preferences of 1989 and 2010

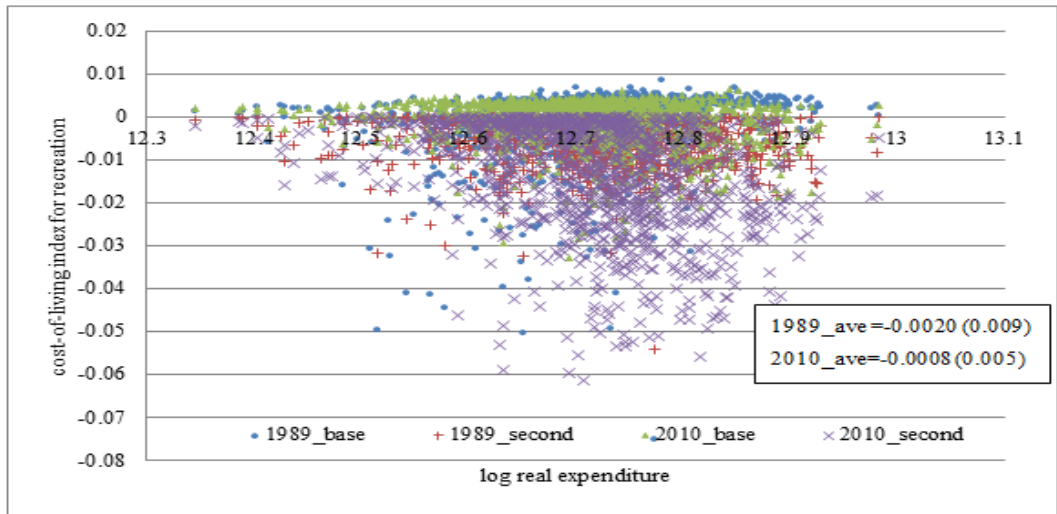**Figure-4.8.** Cost-of-living index for education in base preferences of 1989 and 2010

Figure-4.9. Cost-of-living index for recreation in base preferences of 1989 and 2010
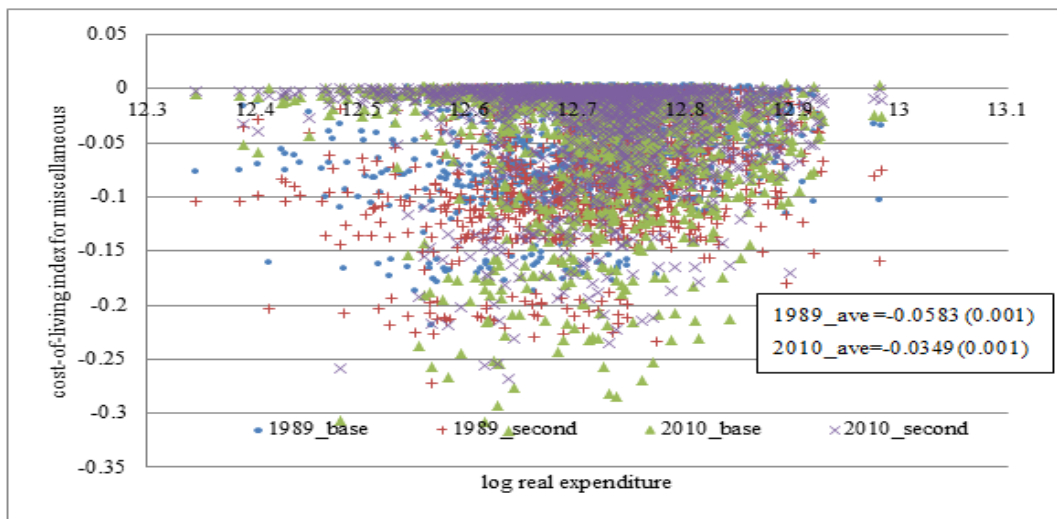

Figure-4.10. Cost-of-living index for miscellaneous in base preferences of 1989 and 2010

6. CONCLUDING REMARKS

In this study, we measured the changes in the cost-of-living index in Japan comparing it with calculations using two base preferences. We found that the cost-of-living index was shifted upward by price changes despite the existence of a protracted deflation. That is, we confirmed an increase of consumer surplus with a change in consumer expenditures, prices, and demographics. In addition, in the base year of 2010, the cost-of-living indices were stably distributed around the average value and its deviations were small, unlike the case when the base year is 1989. The change in the cost-of-living index reduced the gap in the cost-of-living index between every range of log real expenditure.

The cost-of-living index is derived from the budget shares of the base preference and substitution effects, and the change in the index can be expressed by the change in these two effects. In our analysis, in almost all 10 goods, the majority of these changes could be explained by the substitution effect. That is, we found that Japanese households must substitute greatly for cross-price goods in the face of price changes, and this was the main factor behind the increase in consumer surplus. We also found that allowing errors to equal unobserved heterogeneity had little impact on parameter estimates as well as the measures of the cost-of-living index.

The long-term downturn in prices was problematic for the Japanese economy, but we found that the cost-of-living index, itself, increased over approximately 20 years studied in this research. We also showed that a difference in the level of consumer surplus derived from differences in log real expenditure decreased with the base year as 2010. This can be considered as both an increase of consumer surplus in the low-range in log real expenditure and the stagnation of growth of consumer surplus in the high-range in log real expenditure. On the other hand, we can consider that there is a possibility that consumer surplus has reached a plateau due to long-term deflation. If we can expect a permanent rise in price levels because of the policy effects of Abenomics, a further increase in consumer surplus levels will appear. By the extension of data periods in the future, it will be able to verify the effects of the policies under Abenomics.

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